Aberrations and allowances for errors in a hemisphere solid immersion lens for submicron-resolution photoluminescence microscopy

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We have analyzed the aberrations and allowances for an aspheric error, a thickness error, and an air gap in a hemisphere solid immersion lens for photoluminescence microscopy with submicron resolution beyond the diffraction limit, where light with large ray angle and the effect of an evanescent field play important roles. © *1999 American Institute of Physics*. [S0021-8979(99)06209-X]

Various applications of a solid immersion lens (SIL)^{1,2} have recently been developed, for example, in high-density optical storage,^{3,4} near-field-scanning-optical-microscope probes,⁵ photoluminescence (PL) microscopy of semiconductor with high-collection efficiency, in vacuum, and at low temperatures.^{6–8} Its basic principle is the same as that of oil-immersion microscope and hence is well known, while all-solid oil-free operation enables such advantages as no contamination of specimen and usefulness in vacuum, and at low temperature. The light transmission properties through a solid-air-solid surface has particularly been studied by Guerra, who realized high-resolution three-dimensional microscopy of various solid surfaces as photon-tunneling microscopy.^{9,10}

When one prepares and uses a SIL in practice, deviation from the ideal design degrades microscopic images. Thus, it is important to know aberrations in an imperfect SIL and its allowances, though their analytic expressions or similarly useful data have so far been reported only partly, for example, under the para-axial approximation.³

In this article, we present the aberrations and allowances of hemisphere SILs with typical possible errors in from convenient and useful for the practical design, fabrication, and application of SILs. An aspheric error, a thickness error, and an air gap are examined on the basis of point-spread function (PSF) for PL microscopy without the para-axial approximation, which is important in submicron-resolution microscopy beyond the vacuum diffraction limit. In numerical examples, we mainly discuss a SIL with refractive index n=2 and the radius a=1 mm, setting the refractive index of air to be 1. Other choice of *n* and *a* will be mentioned in the end of this article.

For the ideal hemisphere SIL, there exists no aberration for light focused into the sphere center, since all light rays incident normally to the hemisphere surface converge at the center. The magnification and resolution improvement factors are both *n*. In practice, however, aberrations $W(\theta)$ are incorporated for each plane wave with incident angle θ in the PSF integral as the relative phase $\Phi(\theta) = 2\pi W(\theta)/\lambda$ from that of $\theta = 0$, where λ is the wavelength of light in vacuum. Then, the PSF is expressed as,

$$I(x) \propto \left| \int_0^{\sin \theta_0} J_0 \left(\frac{2 \pi n x \sin \theta}{\lambda} \right) e^{i \Phi(\theta)} \sin \theta d(\sin \theta) \right|^2, \quad (1)$$

where $\sin \theta_0$ and J_0 show the numerical aperture of the objective lens and the Bessel function of order zero, respectively.

Figure 1 shows the three typical errors of a hemisphere SIL, an aspheric error b, a thickness error d, and an air gap h, which we examine in this article.

Since a hemisphere SIL is typically fabricated by grinding a ball lens with refractive index n and radius a into hemisphere, the first possible error is the deviated shape of the original ball lens from the perfect sphere. If we model this aspheric error as the variation of a against θ , which is as large as b, it causes the optical path length variation of b(n - 1), that is the aberration

$$W = b(n-1). \tag{2}$$

One of the easiest convenient conditions on allowances for aberrations is the "quarter-wavelength" condition, $W < \lambda/4$ or $\Phi < \pi/2$, so that the allowances for the aspheric error *b* is given by,

$$b < \frac{\lambda}{4(n-1)}.$$
(3)

The allowed *b* is smaller for larger *n* or for smaller λ . For a SIL with n=2, the spheric surface accuracy of an original ball lens should be within $\lambda/4$, which is achieved in some commercial ball lenses.

The second possible error is the thickness error of a SIL, where the excess thickness d (d < 0 for insufficient thickness) from a for hemisphere introduces the spherical aberration at the bottom surface in the SIL. For $d/a \ll 1$, the analytic expression of W is obtained as,

$$W = \frac{d^2}{a}n(n-1)\left(1 - \cos\theta - \frac{1}{2}\sin^2\theta\right) \text{ (for all } \sin\theta\text{)},$$
(4)

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FIG. 1. The schematic of the typical errors in a hemisphere SIL, an aspheric error b, a thickness error d, and an air gap h.

$$\approx \frac{d^2}{8a} n(n-1)\sin^4 \theta \quad (\text{if } \sin \theta \ll 1), \tag{5}$$

$$\approx \frac{d^2}{2a}n(n-1) \quad (\text{if } \sin\theta \approx 1). \tag{6}$$

The second equation, Eq. (5), under the para-axial approximation is identical with the result by Mansfield.³ The quarter-wavelength condition for the maximum W at $\sin \theta \approx 1$ gives the allowances for the thickness error d as,

$$|d| < \left[\frac{2a\lambda}{n(n-1)}\right]^{1/2}.$$
(7)

Note that the dependence of *W* on *d* has no linear term, but starts from the square term of *d*, so that the processing error of thickness for a hemisphere SIL is tolerated proportionally to the square root of the radius *a*. This is different from the other type of SIL with the shape of superhemisphere, or Weierstrass sphere, which we discuss in a separate article.¹¹ If we use a SIL with a=1 mm and n=2 for $\lambda = 600 \text{ nm}$, we can tolerate the thickness error $d=\pm 24 \mu \text{m}$ from a perfect hemisphere. This is far larger than the fabrication accuracy of the order of 1 μm .

The third possible error is an air gap, or the distance h between a SIL and an object. Here, we discuss the simplest case that the object is a point light source on a surface of a substrate with the same refractive index n as the SIL. Then, to be added in the PSF integral is the electric-field transmission coefficient $t(\theta) = |t| \exp i\delta$ through an air gap of thickness h between solid with refractive index n, which is described in the textbook by Born and Wolf.¹² The aberration W is given by,

$$\Phi(\theta) = \frac{2\pi}{\lambda} W(\theta) = \left[\delta(\theta) - \delta(0) \right] - \frac{2\pi}{\lambda} nh(\cos\theta - 1).$$
(8)

Since *t*, and hence δ , are different for *p* and *s* polarizations due to multiple reflection at the air-solid and solid-air interfaces, *W* is also polarization dependent. We numerically calculated aberrations *W* for *p* and *s* polarizations in particular case of $h = \lambda$ and n = 2, which are shown by thick solid and broken curves in Fig. 2(a).

Also shown by a thin curve is the calculated result with single-path approximation for incident angles below the critical angle $\theta_c = \sin^{-1}(1/n)$, for which we have the analytic expression as,



FIG. 2. (a) Calculated wave aberration W/λ as a function of light incident angle θ plotted by thick solid and broken curves for *p* and *s* polarization light, respectively $(h=\lambda, n=2)$. Also shown by a thin curve is the calculated result [Eq. (9)] with single-path approximation for θ below the critical angle $\theta_c = \sin^{-1}(1/n)$. (b) Amplitude factor $|t(\theta)|$ as a function of light incident angle θ for *p* and *s* polarizations (solid and broken curves) with varied parameter of $h < \lambda$ (*n*=2).

$$W = \frac{h}{n} \sqrt{1 - n^2 \sin^2 \theta} - nh(\cos \theta - 1)$$

(single-path approx., for $\sin \theta < 1/n$) (9)

$$\approx -\frac{n(n^2-1)}{8}h\sin^4\theta \quad (\text{if } \sin\theta \ll 1). \tag{10}$$

This is exactly the same as that calculated with geometrical optics, and Eq. (10) is identical to the previously reported result.³ However, we should note in Fig. 2(a) that Eq. (9) obtained with single-path approximation underestimates W. In addition, W is proportional to h in the single-path approximation, while it is not the case with the effect of multiple reflection. When an incident angle is larger than the critical angle, it is essential to estimate the aberration by means of wave optics.

Note that W/λ for $h = \lambda$ is as small as 1/4. For $h < \lambda$, W is within the quarter-wavelength condition, and hence to be neglected in the evaluation of resolution.

On the other hand, the amplitude factor $|t(\theta)|$ is important in $h < \lambda$, because $|t(\theta)|$ is sensitive to h for $\theta > \theta_c$, as shown in Fig. 2(b). The factor $|t(\theta)|$ essentially reduces the integrated region of θ in the PSF integral and hence limits the effective numerical aperture instead of sin θ_0 .

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FIG. 3. Effective numerical aperture $NA_{\text{eff}}=0.51\lambda$ /FWHM as a function of h/λ for several values of *n*, where FWHM is the full-width of half-maximum of the PSF (sin $\theta_0=1$).

Approximating $|t(\theta)|$ as the average of those of p and s polarizations, which is not critical in the present discussion, we have calculated PSF with varied parameters of an air gap h and n for sin $\theta_0=1$. The spatial resolution is characterized by the full-width of half-maximum (FWHM) of the PSF. In Fig. 3, we plot the effective numerical aperture NA_{eff} defined as $NA_{\text{eff}}=0.51\lambda$ /FWHM as a function of h/λ for several values of n. Note h is always scaled by λ in t, and hence in Φ and in NA_{eff} .

We should first point out in Fig. 3 that NA_{eff} decays rapidly from the ideal value of *n* to 1 for increased h/λ . For subwavelength high-resolution imaging, we need tight contact between a SIL and an object below the spacing of about 0.1 λ .

We point out secondly that the decay of NA_{eff} for increased h/λ is faster for higher *n*. Though higher *n* makes better NA_{eff} , it becomes increasingly important to keep *h* small.

Finally, we comment on other aberrations in a hemisphere SIL.

There is no chromatic aberrations in a hemisphere SIL, though there exists group-velocity dispersion proportional to *a*, which is relevant in an ultrafast time-resolved experiment.

The field of view of a hemisphere SIL limited by the aberration due to the field curvature has been derived by Mansfield as,²

$$W = \frac{r^2}{2a}n(n-1)\sin^2\theta \text{ (for all }\sin\theta), \tag{11}$$

where *r* is the field-of-view radius, or the transverse distance of the observed point from the optical axis, of the SIL. The quarter-wavelength condition for the maximum *W* at $\sin \theta \approx 1$ gives the field-of-view diameter 2r as,

$$2r < \left[\frac{2a\lambda}{n(n-1)}\right]^{1/2},\tag{12}$$

which is similar to the criterion for *d*. Using the same parameter of a = 1 mm, n = 2, and $\lambda = 600$ nm as before, we obtain the field-of-view diameter 2r of about 24 μ m, which is large enough for various applications. Note that the field of view is proportional to the square root of *a* of a SIL.

To summarize, we have shown expressions and numerical data of aberrations and allowances for typical errors in a hemisphere SIL, which is important to make and use the SIL. Though a SIL with higher n can make better resolution, allowances for aspheric error b, thickness error d, air gap h and field of view r become narrower to keep the high resolution. On the other hand, a SIL with larger a has wider allowances for thickness error d and the field of view r proportionally to the square-root of a, while those for aspheric error b and air gap h are unchanged. To use a SIL with larger a, attention should also be paid to the heat capacity, the bottom-surface flatness, and the group-velocity dispersion of the SIL, and to the working distance of an objective lens.

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